

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [2016-19]

B.A./B.Sc. THIRD SEMESTER (July – December) 2017

Mid-Semester Examination, September 2017

Date : 14/09/2017

MATH FOR ECONOMICS (General)

Time : 12 noon – 1 pm

Paper : III

Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

Answer **any two** from **Question Nos. 1 to 3** :

[2×5]

1. Show that the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards in $-1 < x < 0$ and $2 < x < \infty$ but concave downwards in $-\infty < x < -2$ and $0 < x < 1$ and $x = -2, 0, 2$ are points of inflection.

[5]

2. a) Show that the curve $y = e^x$ is at every point convex to the foot of the ordinate of that point.

[2]

b) Prove that arbitrary union of open subsets of $\mathbb{R}^2$ is open. Give an example to show that arbitrary union of closed set is not closed.

[3]

3. a) Examine the existence of the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$, $x \neq 0$, $y \neq 0$.

[2.5]

b) Show that the following function f is continuous at $(0,0)$.

$$f(x, y) = \begin{cases} (ax + by) \sin\left(\frac{x}{y}\right), & y \neq 0 \\ 0, & y = 0 \text{ and for all } x \end{cases} \quad (a, b \in \mathbb{R})$$

[2.5]

4. Answer **any one** question :

[1×3]

a) Determine the set of limit point of $S = \left\{ \left(\frac{1}{m}, \frac{1}{n} \right) : m, n \in \mathbb{N} \right\}$. Is S closed set? Justify your answer.

[3]

b) Define point of inflection. Show that the curve $y = \sin\left(\frac{x}{a}\right)$ has a point of inflection whenever the curve crosses the axis of x . (i.e., $y = 0$).

[1+2]

Group – B

5. Answer **any three** questions :

[3×2]

a) Determine the order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx} \right)^2 \right] \frac{d^3y}{dx^3} - 3 \frac{dy}{dx} \left(\frac{d^2y}{dx^2} \right)^2 = 0$.

b) Show that $\log_e(xy) = cx$, where c is an arbitrary constant, is a primitive of the differential equation $x \frac{dy}{dx} + y = y \log_e(xy)$.

c) Find the non-zero value of 'n' for which the differential equation $(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)dy = 0$ is exact.

d) Reduce the following equation $\sin y \frac{dy}{dx} = \cos x (2 \cos y - \sin^2 x)$ to a linear equation.

6. Solve **any two** of the following :

[2×3]

a) $\log_e \left(\frac{dy}{dx} \right) = ax + by$

b) $xy \frac{dy}{dx} - y^2 = (x + y)^2$

c) $(2xy + e^x)ydx - e^x dy = 0$

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